

intersecting lines are preserved and circles will transform into other circles. The load reflection coefficient can be written as  $|\Gamma_L| e^{i\arg \Gamma_L}$ , where for a passive load  $|\Gamma_L| \leq 1$ . At some output reference plane, the insertion of varying amounts of matched lossless line will vary the angle of  $\Gamma_L$  so that the locus of  $\Gamma_L$  will be a circle centered at the origin and with a radius of  $|\Gamma_L|$ . The locus of the input reflection coefficient will also be a circle which will not in general be centered at the origin of the  $\Gamma_{in}$  plane. If  $|\Gamma_L| = 1$ , the  $\Gamma_{in}$  circle can be used to measure [2], [3] the scattering coefficients of the two-port. This circle may be referred to as the loss circle of the two-port. It is noted that if the load is dissipative (i.e.,  $|\Gamma_L| < 1$ ), the  $\Gamma_L$  circles will be concentric with the unity circle and will have smaller radii. The  $\Gamma_{in}$  circles will lie within the transformed unity circle, although the centers of the circles will not coincide but lie [4] on a straight line connecting the iconocenter and the origin of the  $\Gamma_{in}$  plane. If the angle of  $\Gamma_L$  is held constant (modulo  $\pi$ ), this will describe a diameter of the unity circle in the  $\Gamma_L$  plane. This will map into the  $\Gamma_{in}$  plane as arcs of circles orthogonal to transformed constant  $|\Gamma_L|$  circles. Since all of the diameters in the  $\Gamma_L$  plane intersected at the origin, all of the arcs in the  $\Gamma_{in}$  plane will intersect at the iconocenter.

#### ACTIVE NETWORKS AS LOADS

If the restriction that the  $|\Gamma_L| \leq 1$  is removed, corresponding to a source of power or negative resistance at the output, the extension of the theory follows logically. If  $|\Gamma_L|$  is a constant and greater than unity, the locus will be a circle with a radius greater than unity (i.e., outside the Smith Chart). It will be mapped into a circle which may or may not be all or in part outside the Smith Chart. If no part of the circle is outside the unity circle, an observer at the input port could not tell that there was an active element at the output. If part (or all) of the circle is outside of the chart the observer might (or would) see power coming out of the two-port's input, depending on the phase angle of  $\Gamma_L$ . If the angle of  $\Gamma_L$  is held constant (modulo  $\pi$ ), the extension of the diameter in the  $\Gamma_L$  plane will map into the complete orthogonal circle as the magnitude of  $\Gamma_L$  varies from zero to infinity. It should be noted that the radii of the orthogonal circles will vary and one will have an infinite radius (a straight line). For every phase angle of  $\Gamma_L$  there will be a maximum  $|\Gamma_{in}|$ . Examining (2) it can be seen that if  $s_{22}\Gamma_L = 1$  the input reflection coefficient will become infinite. Since  $b_1/a_1$  is infinite and  $a_1$  is presumed to be finite,  $b_1$  is infinite or infinite power is coming out of the input of the two-port. The value of  $\Gamma_L$  equal to the reciprocal of  $s_{22}$  for infinite power can also be obtained from (1b) as the power at the output becomes infinite. It is noted that letting  $\Gamma_L$  approach infinity will not represent infinite power except when  $s_{22} = 0$ .

#### ACTIVE NETWORKS IN THE TWO-PORT

Returning to the passive load, it might appear that if  $|s_{22}|$  was equal to unity there was the possibility of obtaining infinite power from a passive two-port. From the

conservation of energy in the passive two-port  $|s_{12}|^2 + |s_{22}|^2 \leq 1$ . Therefore  $s_{12} = 0$ . Similarly, it can be shown by taking the equality<sup>1</sup> again  $|s_{11}| = 1$ . When these values are substituted in (2),  $\Gamma_{in} = s_{11}$ . The two-port has been broken into two disjoint one-ports (no transmission between the two).

If the two-port contained some active elements the scattering coefficients could have any value. It is probable that the active elements will alter the reciprocity relationship  $s_{12} = s_{21}$ . However an equivalent reciprocal<sup>2</sup>  $s_{12}'$  could be determined by the Deschamps method  $s_{12}' = \sqrt{s_{12}s_{21}}$ . Therefore it is clear that active elements can be handled whether they appear in the load or the two-port.

#### REPRESENTATION OF MICROWAVE CIRCUITS

The bilinear transformation may be written as  $\Gamma_{in} = T(\Gamma_L)$  where  $\Gamma_{in}$  and  $\Gamma_L$  represent the input and the output of the two-port, while the transformation  $T$  describes the two-port uniquely. Any  $\Gamma_{in}$  circle can be obtained from an infinite combination of  $T$ 's and  $\Gamma_L$ 's. The transformations  $T$  can be considered as belonging to three distinct types of transformations depending on whether  $|\Gamma_L|$  is greater than, equal to, or less than unity. For a given loss circle a set of scattering coefficients can be determined. Only if  $|\Gamma_L| = 1$ , the determined scattering coefficients will be the actual coefficients of the network. However, an observer at the input is unable to distinguish how the given loss circle is obtained and he can represent the two-port and the load as an "equivalent" two-port with a purely reactive load. If the load is purely reactive the "equivalent" network becomes the actual network.

If the other two parameters,  $\Gamma_L$  and  $T$ , are held constant (separately), added information may be found out about the behavior of microwave circuits. If the transformation  $T$  is held constant, the two-port is invariant, and the previous discussion regarding the transformation of  $\Gamma_L$  circles to  $\Gamma_{in}$  circles is applicable. The converse of the previous statement is also true since inverse transformation is also bilinear. If the load reflection circle is held constant, varying  $\Gamma_{in}$  will determine the transformation or the network.

Therefore it can be seen by the extension of the reflection coefficient chart that it is possible to represent any two-port and load at the input by another two-port with a purely reactive termination. Therefore a prescription of the  $\Gamma_{in}$  and  $\Gamma_L$  circles will determine a network. This description reduces to that of Deschamps when the  $\Gamma_L$  circle is the unit circle. The graphical method gives a clear geometric picture of the behavior of a given two-port in terms of input and output reflection coefficient loci.

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#### Characteristics of a Ferrite-Loaded Rectangular Waveguide Twist\*

The Faraday effect in a straight rectangular waveguide, a section of which is completely filled with a ferrite material subjected to an axial magnetic field, has been described by Du Pré,<sup>1</sup> who states that, owing to the presence of a medium of dielectric constant and permeability greater than those of air, modes other than the usual  $TE_{10}$  mode may be propagated in the ferrite-filled section. In particular, the  $TE_{01}$  mode whose electric vector is perpendicular to the narrow dimension of the guide may be supported. If, owing to Faraday rotation, the  $TE_{10}$  mode is converted to the  $TE_{01}$  mode, propagation cannot take place beyond the ferrite-filled section. Experimentally this was confirmed by Du Pré who observed a minimum of transmitted power for  $90^\circ$  rotation. Similar results were obtained in this laboratory with a straight rectangular guide loaded with a cylindrical ferrite specimen the ends of which were tapered for matching purposes as shown in Fig. 1(a). For a given axial magnetic field the reduction of transmitted power was largest with the specimen in the center of the guide, but, as might have been expected, no nonreciprocal effects were observed. A twisted rectangular waveguide section, however, loaded with the same specimen, exhibited nonreciprocal characteristics. In the experiment the sample was mounted centrally midway between the flanges of a  $90^\circ$  commercial  $0.4 \times 0.9$  inch twist and an axial magnetic field was applied as shown in Fig. 1(b).

For constant incident power the transmitted power varied with both the magnitude and the direction of the magnetic field. With the particular nickel-cobalt ferrite used, nonreciprocal behavior was most pronounced at around 8900 mc where, at the optimum value of field current, reversal of the magnetic field caused a reduction of

<sup>1</sup> If  $|s_{12}|^2 + |s_{22}|^2 < 1$ , it would be impossible for  $|s_{22}|$  to be unity and therefore  $|s_{22}\Gamma_L| < 1$ .

<sup>2</sup> Only as far as an observer at the input is concerned.

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<sup>1</sup> F. K. Du Pré, "Experiments on the microwave Faraday and Cotton-Mouton effects," *Proc. Symp. on Modern Advances in Microwave Techniques*, New York, N. Y., pp. 205-213; November, 1954.

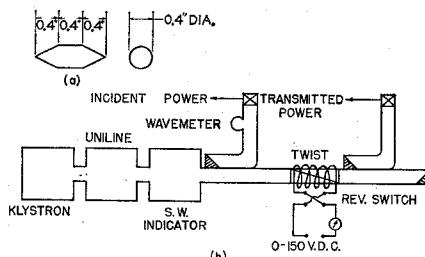


Fig. 1—(a) Tapered cylindrical ferrite specimen; (b) block diagram of experimental equipment.

the transmitted power by more than 10 db. The power ratio on reversal of the magnetic field is shown in Fig. 2 at a constant value of magnetizing current for the frequency range of 8600–9700 mc and in Fig. 3(a) at a constant frequency of 8900 mc with the magnetizing current variable. The insertion loss was approximately equal to the power ratio over the range investigated. Conditions for attenuation in the forward direction are shown schematically in Fig. 3(b).

The voltage standing wave ratio of the ferrite-loaded twist was about 2.5 and was

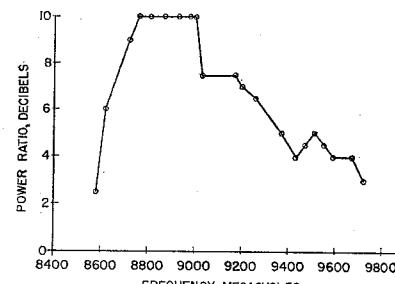


Fig. 2—Transmitted power ratio on reversal of magnetic field at constant magnetizing current ( $H = 160$  oersteds).

not affected appreciably by changes of frequency or magnetic field. The corresponding reduction of incident power of approximately 1 db was taken into account in plotting the above data.

The field configuration in the loaded rectangular twist does not lend itself to simple analysis, particularly since higher modes are likely to be excited, because of the discontinuity presented by the ferrite specimen. However, it is conceivable that a combination of  $TE_{10}$  and  $TE_{01}$  modes could result in

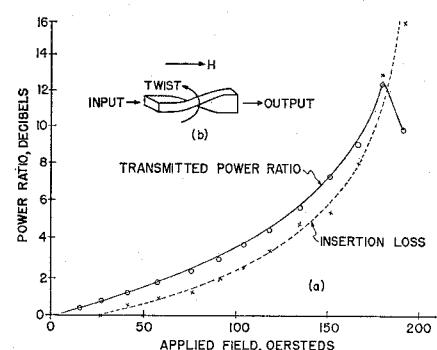


Fig. 3—(a) Transmitted power ratio and insertion loss at constant frequency ( $f = 8900$  mc); (b) conditions for attenuation in forward direction.

a circularly polarized component which in conjunction with the axial magnetic field causes the effects described.

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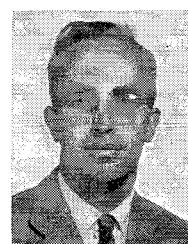


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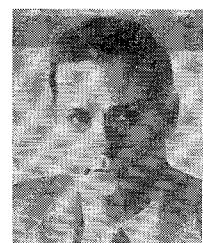


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